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Questions and Answers

A. V. Balakrischnan (*J. Stat. Phys.* 1:227, 1969) raised the question of what is the correct derivative of a function $x(t)$ defined as a definite integral of some statistical function $n(t)$. I am much struck by the great resemblance between Balakrischnan's example and the introduction to a recently published book by H. P. McKean, Jr. (*Stochastic Integrals*, Academic Press, New York, 1969). Indeed, if I may quote the opening sentences:

> "This book deals with a special topic in the field of diffusion processes: differential and integral calculus based upon the Brownian motion. Roughly speaking, it is the same as the customary calculus of smooth functions, except that in taking the differential of a smooth function f of the 1-dimensional Brownian path $t \to b(t)$, it is necessary to keep two terms in the power series expansion and to replace $(db)^2$ by dt:
>
> $$df(b) = f(b)db + \tfrac{1}{2}f''(b)(db)^2 = f'(b)db + \tfrac{1}{2}f''(b)dt,$$
>
> or, what is the same,
>
> $$\int_0^t f'(b)db = f(b)\,|_0^t - \tfrac{1}{2}\int_0^t f''(b)ds.$$
>
> This kind of calculus exhibits a number of novel features; for example, the appropriate exponential is $e^{b-t/2}$ instead of the customary e^b... ."

It would seem that Balakrischnan's logical query has not only been anticipated but has already been expanded into a minor discipline.

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